

Calculus Methods (Speedy Study Guides)

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CALCULUS METHODS

Exact Integrals as Limits of Sums

Using the definition of an integral, we can evaluate the limit as n goes to infinity. This technique requires a fairly high degree of familiarity with summation identities. This technique is often referred to as evaluation "by definition," and can be used to find definite integrals, as long as the integrands are fairly simple. We start with definition of the integral:

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{i=1}^n f(x_i^*)$$

$$= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{i=1}^n f\left(a + i \frac{b-a}{n}\right).$$

Then picking x_i^* to be $x_i = a + i \frac{b-a}{n}$ we get:

In some simple cases, this expression can be reduced to a real number, which can be interpreted as the area under the curve if $f(x)$ is positive on $[a, b]$.

Example 1

For $\int_0^2 x^2 dx$ by writing the integral as a limit of Riemann sums,

$$\int_0^2 x^2 dx = \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{i=1}^n f(x_i^*)$$

$$= \lim_{n \rightarrow \infty} \frac{2-0}{n} \sum_{i=1}^n f\left(\frac{2i}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \left(\frac{2i}{n}\right)^2$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \frac{4i^2}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{8}{n^3} \sum_{i=1}^n i^2$$

$$= \lim_{n \rightarrow \infty} \frac{8}{n^3} \frac{n(n+1)(2n+1)}{6}$$

In other cases, it is even possible to evaluate indefinite integrals using the formal definition. We can define the indefinite integral as follows:

$$\int f(x) dx = \int_a^x f(t) dt = \lim_{n \rightarrow \infty} \frac{x-a}{n} \sum_{i=1}^n f(t_i)$$

$$= \lim_{n \rightarrow \infty} \frac{x-a}{n} \sum_{i=1}^n f\left(a + \frac{(x-a) \cdot i}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{x-a}{n} \sum_{i=1}^n f\left(\frac{x-i}{n}\right)$$

Example 2

Suppose $f(x) = x^2$, then we can evaluate the indefinite integral as follows:

$$\int_0^x f(t) dt = \lim_{n \rightarrow \infty} \frac{x-a}{n} \sum_{i=1}^n f\left(\frac{x-i}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{x}{n} \sum_{i=1}^n \left(\frac{x-i}{n}\right)^2$$

$$= \lim_{n \rightarrow \infty} \frac{x}{n} \sum_{i=1}^n \frac{x^2 - 2xi + i^2}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{x^3}{n^3} \sum_{i=1}^n 1 - \frac{2x}{n^2} \sum_{i=1}^n i + \frac{1}{n} \sum_{i=1}^n i^2$$

$$= \lim_{n \rightarrow \infty} \frac{x^3}{n^3} \frac{n(n+1)(2n+1)}{6} - \frac{2x}{n^2} \frac{n(n+1)}{2} + \frac{1}{n} \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n \rightarrow \infty} \frac{x^3}{n^2} \frac{n(n+1)(2n+1)}{6} - \frac{2x}{n} \frac{n(n+1)}{2} + \frac{1}{n} \frac{n(n+1)(2n+1)}{6}$$

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(Mr. Sterling Hane)

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